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VIBRATION TEST FORCE LIMITS DERIVED FROM FREQUENCY SHIFT METHOD

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During the past three years, input force limiting has been utilized in ten JPL vibration tests (Refs. 1 & 2) to prevent overtesting of flight hardware. In force limited vibration tests, the shaker force is limited to the predicted maximum flight force, plus a specified margin. Force limiting provides a rational and economical solution to the overtesting problem associated with hard mounting of the test item. The two conventional approaches to the overtesting problem are either to develop "bullet proof" hardware, which is very expensive, or to limit the responses of the test item to predicted flight levels, Input force limiting is in theory equivalent to response limiting, but force limiting is often more convenient (critical response locations are sometimes numerous and not accessible) and less dependent on the details of payload models. Implementation of force limiting requires: derivation of a force specification (analogous to that for acceleration), a vibration test fixture to accommodate force sensors, and shaker operation with dual control of both acceleration and force.

This paper focuses on the derivation of force specifications. There are essentially no flight data, and little system test data, on the forces at mounting structure and payload interfaces. Force specifications are therefore derived using structural impedance data on mounting structures and payloads from tap tests and shaker tests with force gages, as well as from finite ellement analyses. In previous JPL applications of force limiting, the force specifications have been derived using one of two methods (Ref. 3): blocked force or two-degree-of-freedom system (TDFS) mean-square response. These methods were adequate for previous testing applications, because a conservative approach was adopted, but the methods involve some inherent conceptual difficulties and do not adequately represent the contribution of resonant and nonresonant vibration modes to the interface force.

DEVELOPMENT OF FREQUENCY SHIFT METHOD OF PREDICTING FORCE LIMITS

An improved method of deriving force specifications for force limited vibration tests is described. The derivation is one dimensional, that is only one component of the six component force/moment vector and only one point at the source/load interface is considered. Empirical and analytical methods of defining the required effective mass data are not discussed. Much of the rationale for the method was provided by Ref. 4.

For both the coupled source and load configuration of flight and the isolated load configuration of the vibration test, the interface force autospectrum S_{ff} is related to the interface acceleration autospectrum S_{aa} by Eq. 1, which is F=MA for random vibration:

$$S_{ff}(w) = |M_2(w)|^2 S_{aa}(w) \qquad (1)$$

where: M_2 is the load dynamic mass, i.e. the magnitude and phase of the frequency response functions (FRF) of the ratio of the drive point force to acceleration, which is the same for both configurations, The term "dynamic" mass is used to include the complete dynamic response including resonance and stiffness effects,

The application of Eq. 1 to a simple coupled source and load system is illustrated in Fig. 1. The FRF curves in Fig. 1 are derived in Ref. 5 for the simple TDFS shown in the upper right hand of Fig. 2 for the case of identical oscillators and unit excitation, Fig. la shows the magnitude of the load dynamic mass, which peaks at the load natural frequency f_o with an amplitude Q times the input, (The amplification factor Q is the reciprocal of twice the critical damping ratio.) Fig. 1 b and c show the magnitude of the coupled system interface acceleration and force, respectively. Eq. 1 may be used to calculate the force in Fig. 1 c from the load dynamic mass in Fig. 1 a and the acceleration in Fig. 1 b. For example, applying Eq. 1 at the 0.62 Hz coupled system resonance frequency in Fig. 1, the load mass of approximately 1.6 times the peak acceleration of 50 equals the peak force of 80, and at 1.62 Hz the load mass of approximately 0.6 times the peak acceleration of 8 equals the peak force of 5. Notice from Fig. 1 that the interface force and acceleration peak at the same frequencies, i.e. the coupled system natural frequencies. This is shown to be a general result.

As a first example of the improved method, the maxirnurn force for the simple TDFS with different masses shown in Fig. 2 is calculated. For this TDFS, the maximum response anti maximum force are for the case where the oscillator frequencies are identical, i.e. the dynamic absorber (Ref. 6). The characteristic equation for a dynamic absorber (Ref. 7) is used to calculate the coupled system resonance frequencies for the TDFS in Fig. 2:

$$\beta^2 = (1+u/2) \pm (u+u^2/4)^{0.5} \tag{2}$$

where: β is the ratio of a coupled system resonance frequency to the uncoupled resonance frequency of the load oscillator and u is the ratio of load to source masses M_2/M_1. The peak in the interface force spectrum normalized by the peak in the acceleration spectrum at each of the two resonance frequencies is calculated from the magnitude squared of the load dynamic mass via Eq. 1, which for the TDFS in Fig.2 is:

$$S_{ff}/(S_{aa}\, M_2{}^2) = (1+\beta^2/Q^2)/[(1-\beta^2)^2+\beta^2/Q^2]. \tag{3}$$

which is plotted in Fig. 2 against the ratio u of the load mass M_2 to the source mass Ml. (The greater of the values of Eq. 3 at β_+ and β_- is used.) For small values of the mass ratio, the load has little effect on the source and the maximum normalized force asymptote is Q^2. (Previous force limits based on the mean-square response of a simple TDFS have an asympote of $Q^2/2$, Ref. 3). For large values of the mass ratio, the maximum normalized force approaches unity, i.e. there is no amplification regardless of the Q.

The improved method is called the “frequency shift” method because the maximum forces in the coupled system are calculated by evaluating the load dynamic mass at the coupled system, or shifted, resonance frequencies instead of at its peak which is at the uncoupled load resonance frequency (Fig. 1a). The ratio of the coupled system maximum force to the force in a conventional vibration test, the so called ‘(knock down” factor, is equal to the ratio of the load dynamic mass at the shifted frequencies to its peak value at the uncoupled load resonance frequency.

CALCULATION OF FORCE LIMITS FOR RESIDUAL AND MODAL MASS MODEL

To generate a set of parametric curves which may be used to specify force limits for vibration tests, the frequency shift method is applied to a more complex TDFS in which the source and load each have two masses to represent both the residual and modal masses of a continuous system. Figure 3a shows a model of a source and load in which each mode may be represented as an oscillator attached to the connection interface. Derivation of this type of model from a FEM analysis requires normalizing the modes so that the inertial forces equal the reaction forces at the interface, Ref. 8. When this model is excited at the interface with a frequency near the resonance frequency w_n of the n^{th} mode, the model may be simplified to that in Fig. 3b, where m_n is the modal mass of the n^{th} mode and M_n is the residual mass, i.e. the sum of the masses of the n^{th} and all higher resonance frequency modes. Finally, Fig. 3c shows the coupled system model which results from coupling a residual and modal mass model of both the source and load,

The maximum normalized force for the residual and modal mass TDFS in Fig. 3c is calculated similarly to that for the simple TDFS in Fig. 2, i.e. by evaluating the load dynamic mass, at the coupled system resonance frequencies. It is assumed that the acceleration specification correctly envelopes the higher of the two acceleration peaks of the coupled system. The normalization of the maximum force by the maximum acceleration requires accounting for the ratio of the acceleration peaks at the two coupled system resonance frequencies, because in some cases the higher force peak corresponds to the lower acceleration peak. Calculation of the maximum force for the residual and modal mass model also necessitates a tuning analysis, conducted by considering different ratios of the load and source uncoupled resonance frequencies in 3 % increments. The results are presented in parametric tables and curves for different ratios a_s and al of modal to residual mass for the source and load, respectively.

Tables 1 lists the maximum normalized force as a function of the ratio of load to source residual mass M_2/M_1 for different ratios of source and load modal to residual mass, m_l/M_1 and m_2/M_2, for Q equal 50. The maximum normalized forces are rounded to whole numbers and the tuning frequency ratio squared in 16ths, which results in the maximum forces, is identified by the digits to the right of the decimal in Table 1.

In Fig, 4, the maximum force for two of the residual and modal mass model cases of Table 1, a_s and a_l both equal 1.0 and 0.1, are compared with results from Fig. 2 for the simple TDFS. It is anticipated that in future JPL vibration tests, force limits like those in Table 1 for the residual and modal mass model will replace those in Fig. 2 for the simple TDFS model.

ACKNOWLEDGEMENT

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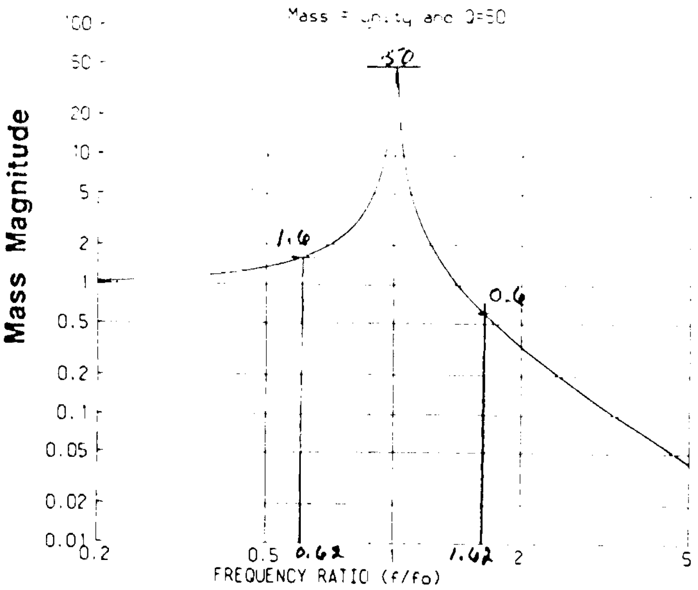
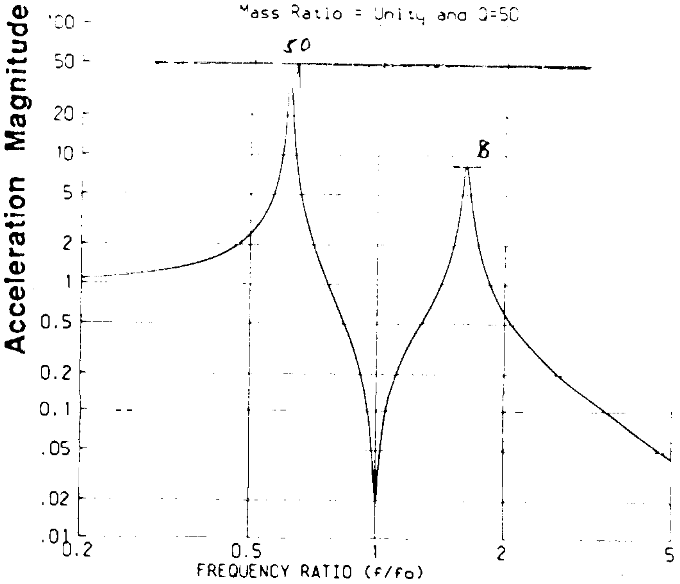
AMPLIFICATION FACTOR (Q)	50									
RESIDUAL MASS RATIO (M2/M1)		0.001	0.003	0.01	0.03	0.1	0.3	1	3	10
RATIO OF MODAL TO RESIDUAL MASS										
Source (m1/M1)	Load (m2/M2)									
1	1	1189.32	592.32	229.32	112.31	53.3	34.27	26.24	23.23	24.22
1	0.7	701.32	380.32	154.32	74.31	37.3	22.28	17.24	15.23	14.22
1	0.5	415.32	245.32	106.32	51.31	26.3	16.28	72.24	11.22	1?.21
1	0.3	?79.32	122.32	60.32	29.31	15.3	10.28	7.25	7.22	7.2
1	0.1	27.32	24.32	?6.32	9.32	6.31	4.29	3.26	3.22	3.2
AMPLIFICATION FACTOR (Q)	50									
RESIDUAL MASS RATIO (M2/M1)		0.001	0.003	0.0?.	0.03	0.1	0.3	1	3	10
RATIO OF MODAL TO RESIDUAL MASS										
Source (m1/M1)	Load (m2/M2)									
0.7	1	1213.27	666.271	295.27	122.27	61.26	36.25	27.23	24.221	23.22
0.7	0.7	699.27	417.27	196.27	84.27	41.26	24.25	18.23	16.22	?5.21
0.7	0.5	408.27	263.27	133.27	59.27	29.26	17.25	12.23	11.21	?1.21
0.7	0.3	171.27	126.27	73.27	35.27	18.26	10.25	8.22	7.21	7.2
0.71	0.1	24.27	22.27	18.27	11.27	7.26	4.26	4.22	3.21	4.?8
AMPLIFICATION FACTOR (0)	50									
RESIDUAL MASS RATIO (M2/M1)		0.0071	0.003	0.01	0.03	0.1	0.3	1	3'	10
RATIO OF MODAL TO RESIDUAL MASS										
Source (m1/M1)	Load (m2/M2)									
0.5	1	1454.24	804.24	335.24	142.24	62.23	40.23	27.22	25.22	23.22
0.5	0.7	825.24	499.24	221.24	95.24	42.23	26.23	19.22	15.22	1s.21
0.5	0.5	472.24	311.24	148.24	66.24	30.23	18.23	13.22	12.21	10.21
0.5	0.3'	195.24	?46.241	80.24	30.241	18.23	11.23	9.21	6.2	7.2
0.5	0.1	27.24	25.24	19.24	12.24	6.24	5.22	4.21	4.19	4.181
AMPLIFICATION FACTOR (Q)	50									
RESIDUAL MASS RATIO (M2/M1)		0.00;	0.003	0.07	0.03	0.1	0.3	1	3	10
RATIO OF MODAL TO RESIDUAL MASS										
Source (m1/M1)	Load (m2/M2)									
0.31	1	1290.27	806.21	390.21	184.21	82.21	42.21	27.22	24.22'	22.22
0.3	0.7	714.21	479.21	247.21	119.21	55.2?	31.21	20.21	17.21	16.2?
0.3	0.5	403.21	269.21	159.21	80.21	37.2?	20.21	13.21	11.21	11.2
0.3	0.31	?65.2?	130.21	81.21	43.21	21.21	13.2	10.2	7.2	7.2
0.3	0.1	24.21	22.21	18.21	12.2?	7.21	6.2	5.19	4.18	4.18
AMPLIFICATION FACTOR (Q)	50									
RESIDUAL MASS RATIO (M2/M1)		0.00?	0.003	0.01	0.03	0.1	0.3	1	3	10
RATIO OF MODAL TO RESIDUAL MASS										
Sows (m1/M1)	Load (m2/M2)									
0.1	1	1006.18	810.18	537.18	330.18	106.18	48.19	29.21	24.21	22.22
o.?'	0.7	525.18	442.10	310.18	197.18	64.18	40.19'	22.2	16.21	16.21
0.1	0.5	284.18	24 S.?8	164.181	121.18	69.18	26.18	15.19	13.2	11.2
0.1	0.3	112.18	103.18	62.18	58.18	36.18	21.18	10.19	9.19	6.19
0.1	0.1	17.18	17.18	15.16	13.18	9.18	6.18	5.16	4.18	4.18

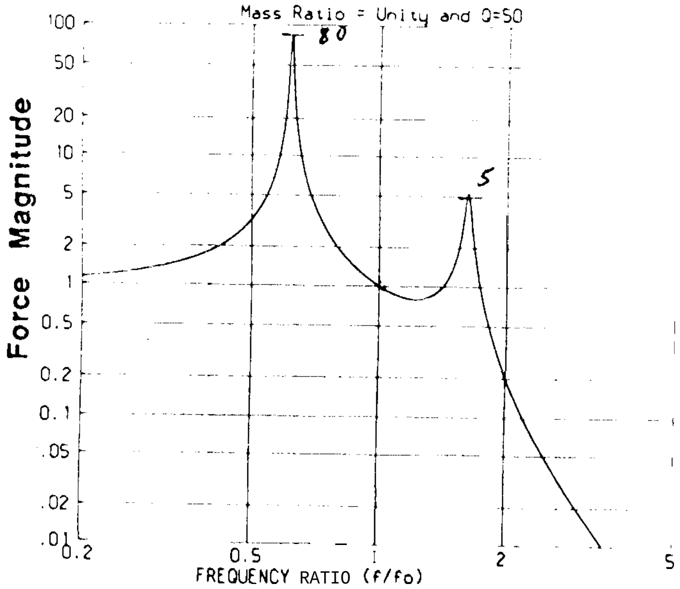
Table 1. **Q=50, Maximum Tuned** Force Spectrum Normalized by Load **Residual Mass** Squared and **Acceleration Test Specification** **(decimals indicate in** sixteenths the **ratio of load to source resonance** frequency **squared for tuned maximum force)**



1a. LOAD DYNAMIC MASS, M_2



Ib. iNTERFACE ACCELERATION, A



1c. INTERFACE FORCE, F

Figure 1. Load Dynamic Mass, Acceleration, and Force Frequency Responses for Identical, Simple TDFS

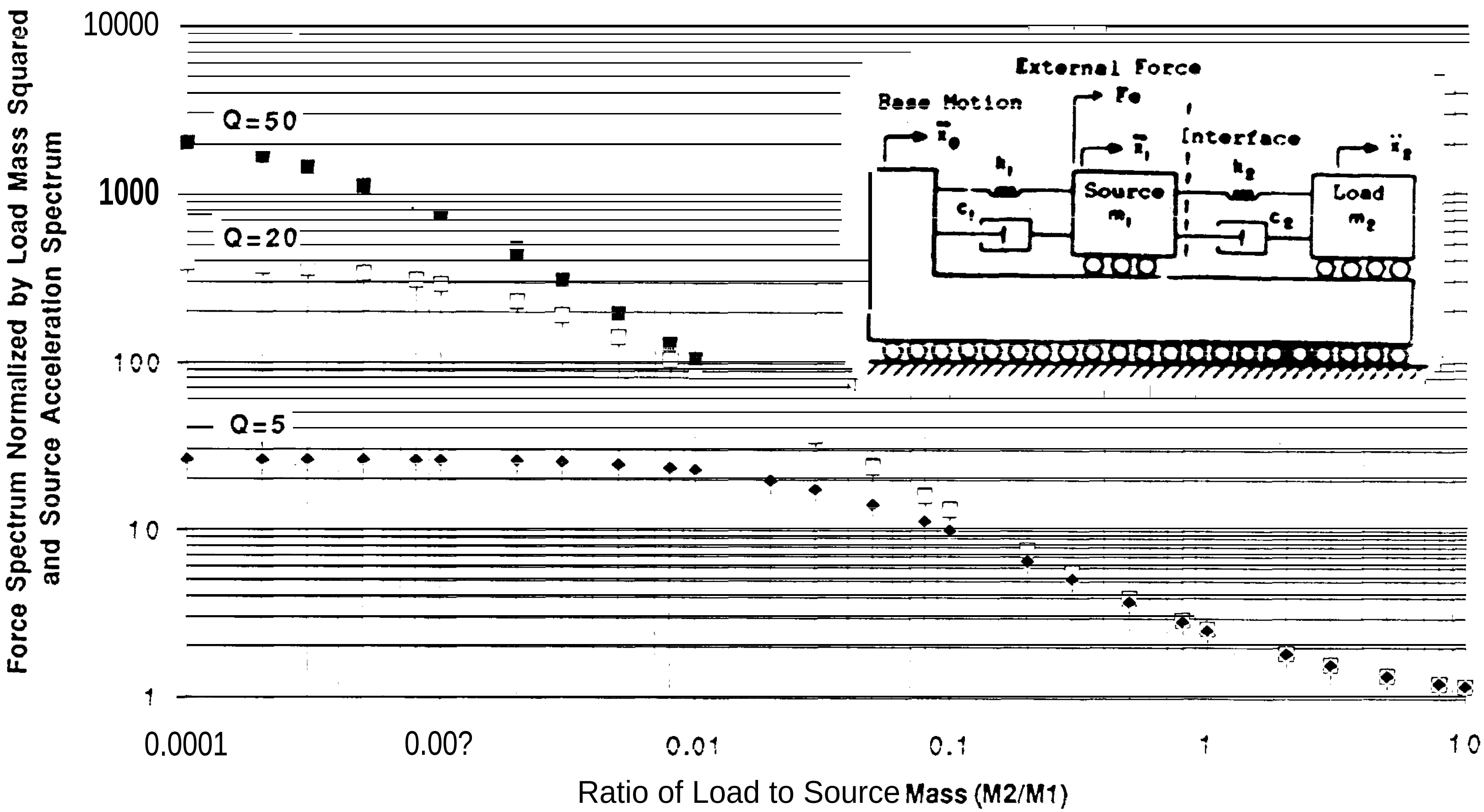


Figure 2. Maximum Normalized Force for Simple TDFS Calculated with Frequency Shift Method

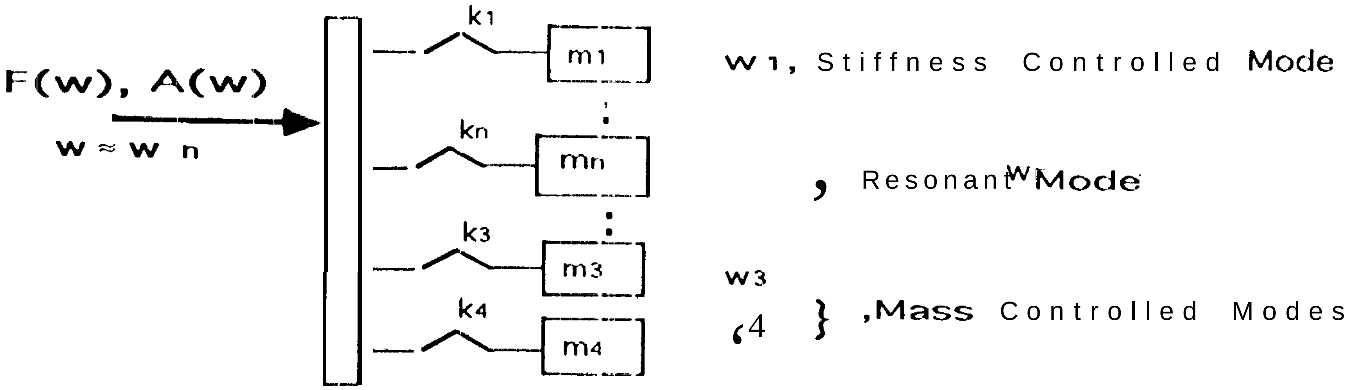


Fig. 3a APPARAGUS PATCH MODEL OF SOURCE OR LOAD

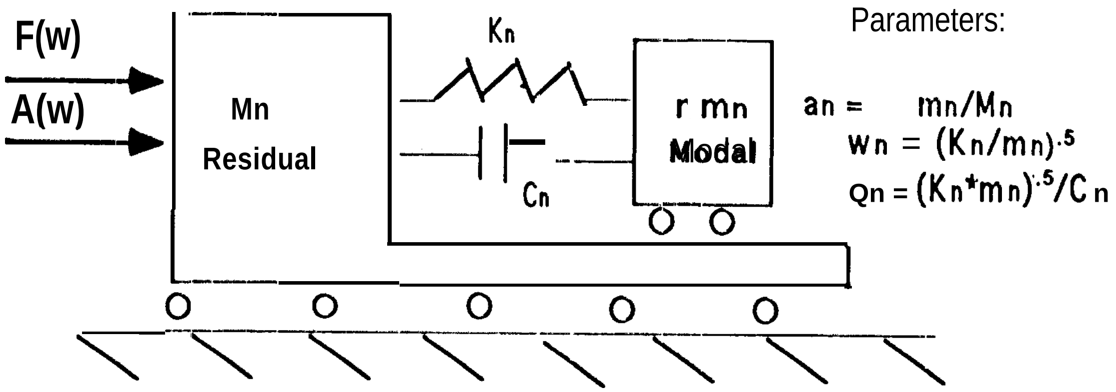


Fig. 3b RESIDUAL AND MODAL MASS MODEL OF SOURCE OR LOAD

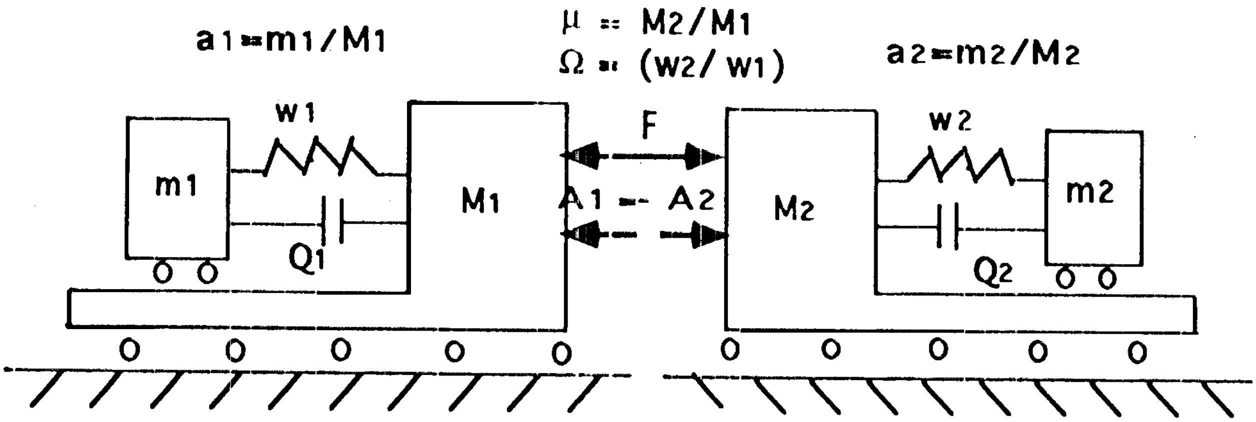


Fig. 3c COUPLED TDFS RESIDUAL AND MODAL MASS MODEL

Figure 4. COMPARISON OF MAXIMUM FORCE SPECTRA FROM ORIGINAL **TDFS** AND FROM **RESIDUAL AND MODAL** MASS **MODELS; Q=50**